Optimization of Bonded Joints

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A systematic procedure for minimizing the elastic shear stress concentration in adhesive lap joints is presented. The proposed method is based upon tapering the adherends in a specific manner to achieve smooth stiffness transitions and uniform shear stresses. Both single and double lap splices are considered, but numerical examples are restricted to the case of double lap joints. Nonisotropic materials and nonoptimum design limitations, such as minimum and maximum thickness adherends, load-line eccentricity, and peel stresses, are treated, and typical results are presented.

Nomenclature

C = load transfer ratio (see Fig. 1)
E = modulus of elasticity
G = shear modulus of elasticity
h = adhesive thickness
L = length of adhesive joint
N = membrane load in adherends
n = number of surfaces in joint; = 1 for single shear, = 2 for double shear
t = thickness of adherends
u = membrane displacement of adherend
w = bending deflection of adherend
x = independent axial joint coordinate
z = independent transverse joint coordinate
γ = shear strain in the adhesive
λ = joint stiffness parameter [see Eq. (12)]
τ = elastic shear stress in adhesive
$(\ )'$ = derivative with respect to x
$\bar{(\ )}$ = average value

Subscripts

a = adhesive
i = inner (for single lap joint, initial)
o = outer (for single lap joint, other)
opt = optimum

Introduction

EXPERIENCE with structural failures has shown that most ruptures initiate at joints where stress concentrations are difficult to avoid. Thus, the rational design of a structural joint can often be the most crucial driver in a strength-critical component. Unfortunately, the typical approach taken in selecting an adequate joint involves an ad hoc trial-and-error procedure, which could require several design and analysis cycles. That is to say, one generally conceives of a bolt pattern, or a bond or weld configuration, and proceeds to analyze and/or test it for strength. This time-consuming procedure results in an overall load-carrying capability that can be greatly reduced from that for the virgin material. Such reduced efficiencies are often caused by stress concentrations that result from local stiffness mismatches among the various elements comprising the joints.

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In this paper we present a bond joint design optimization approach that is much more direct and efficient than the usual hit-and-miss scheme and that can yield improved joint efficiencies. The method depends upon the tailoring of local axial stiffnesses and can yield joint strengths greater than can be achieved without stiffness tailoring. In concept, the method proposed resembles the fully stressed design (FSD) optimization approach of general structural synthesis, in which one aims to load each member to a predetermined stress value. However, there is little similarity between the mechanics of iteratively obtaining an FSD and the proposed approach. The reason for this is that joints involve local, highly redundant stress fields and the FSD procedure works best (i.e., converges rapidly) for structures in which the load path is clearly defined and the redundancy level is low.

While a number of earlier papers discuss the bond optimization problem,[1-4] only Cherry and Harrison's work[4] approaches the problem analytically as does the present scheme. Their paper, however, is limited to joints in single shear and does not consider realistic design constraints. The present work is much more detailed and investigates and also considers the effects of adherend anisotropy, single or double shear, and peel stress.

Detailed numerical results are presented in this paper for the case of double lap shear joints. We hasten to add that the present joint design philosophy is applicable to mechanically fastened joints as well as bonded structures, although such an extension would require specific consideration of the local stiffness of bolts and holes, which is an entire study in itself.

It should be noted that the present work is *not* concerned with the definition of the term "joint efficiency," or the hypothesis that the adhesive stress should be uniform in an optimally designed joint. Although the examples presented are based upon a uniform shear stress as the design objective, the method presented can accept any imposed stress distribution, uniform or nonuniform, for the objective function.

Technical Approach

The proposed procedure is based upon an inverse method, which starts by imposing a desired stress distribution on the mating surfaces in the direction of the primary load transfer path. The adherend elastic membrane equations are then examined to determine the adherend stiffness distribution required to yield a constant stress in the adhesive.

Typical designs for bonded lap joints (in single or double shear) involve the use of uniform thickness adherends over the entire bond length. This leads to severe load-carrying penalties in the brittle elastic adhesive case due to the nonuniformity of stress (both shear and peel) along the bondline. Linear elastic bond analyses indicate that stress concentrations occur at the edges of such joints, and it can be shown that these are due to local stiffness discontinuities. To partially overcome these effects, some designers have resorted to uniformly tapered or step joints. However, the amount of taper or stepping required generally involves a trial-and-error design and analysis procedure.

The present approach for minimizing stress concentrations in the adhesive is to solve the inverse problem, i.e., to first postulate a desired shear stress distribution and then to determine the associated adherend stiffness distributions required to achieve this stress state. This procedure leads to an explicit ideal adherend thickness variation which unfortunately is not always physically achievable because of practical design constraints. In addition, the eccentricity of the bondline from the loadline induces peel stresses in the bond. Since these peel stresses are not explicitly optimized by this procedure, it is necessary to check the final "constrained" design to determine the actual shear and peel stresses.

Referring to the double lap joint shown in Fig. 1, we wish to determine the outer adherend thickness distribution $t_0(x)$ such that the bond shear stresses $\bar{\tau}_{\rm opt}$ are uniform over the entire bond length L. If N is the load (per unit length normal to the figure) applied to the inner member and C is the ratio of load retained within the inner member to N, then the optimum (average) uniform shear stress is given by (see Fig. 2)

$$\bar{\tau}_{\rm opt}=N(1-C)/2L=\text{const} \tag{1}$$

for a bonded joint in double shear, or twice this value for a joint in single shear.

For a linearly elastic adhesive material, the shear stress is related to the shear strain γ_a and adhesive shear modulus G_a by the relationship

$$\bar{\tau}_{\rm opt}=G_a\gamma_a=G_a\left(\frac{\partial u_a}{\partial z}+\frac{\partial w_a}{\partial x}\right) \tag{2}$$

where u_a and w_a are adhesive deflections in the x and z directions, respectively (see Fig. 3).

Assuming a through-the-thickness linear distribution of deflections in the adhesive, we have

$$u_a=u_i+(z/h)(u_o-u_i),\quad w_a=w_i+(z/h)(w_o-w_i) \tag{3}$$

where the subscripts i and o denote inner and outer adherends (or "initial" and "other," for single lap joints) and h is the bond thickness.

Assuming that the adherends obey classical beam theory, then the symmetry of a double lap joint requires that there be no bending deflections in the inner adherend, i.e., $w_i=0$. Substituting Eq. (3) into Eq. (2) and assuming that $w_i=0$, we obtain

$$\bar{\tau}_{\rm opt}=G_a[(u_o-u_i)/h],\quad u_o'=u_i' \tag{4}$$

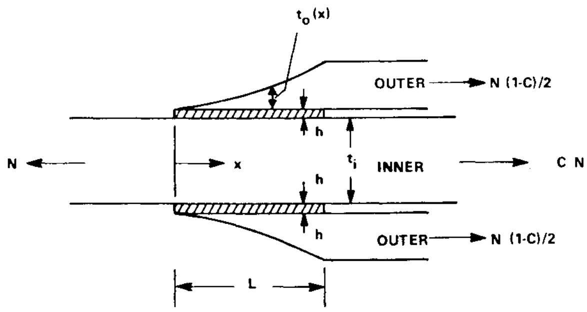

Fig. 1 Double lap bonded joint.

at the surface of the inner adherend, $z=0$. Equations (4) are only approximations for single lap joints but represent the dominant terms contributing to shear stress. The effects of the additional terms upon shear stress have been discussed by Ojalvo and Eidinoff.[5]

Uniform Material Case

Combining Eq. (4) with the uniform adherend stress-strain-extension relations

$$N_i=E_it_iu_i',\quad N_o=E_ot_ou_o' \tag{5}$$

where the combined E and t terms are the adherend membrane stiffnesses, gives

$$N_i/E_it_i=N_o/E_ot_o \tag{6}$$

The horizontal force equilibrium equations for the adherends yield

$$\bar{\tau}_{\rm opt}=N_o'=-N_i'/n \tag{7}$$

where $n=2$ for joints in double shear and 1 for joints in single shear.

Integrating Eq. (7) and making use of the boundary conditions $N_o(0)=0$ and $N_i(0)=N$ yields

$$N_i=N-nx\bar{\tau}_{\rm opt},\quad N_o=x\bar{\tau}_{\rm opt} \tag{8}$$

Substituting Eqs. (1) and (8) into Eq. (6) yields the expression for the optimum-shaped adherend, i.e.,

$$t_o=\frac{E_it_i}{nE_o}\left(\frac{L/x}{1-C}-1\right)^{-1} \tag{9}$$

For the case where $t_i(L)=0$ and $C=0$, t_o remains finite at $X=L$ for the optimum joint. In particular, if t_i is uniformly tapered, so is t_o. Thus, when

$$t_i=t\left(1-\frac{x}{L}\right)$$

then

$$t_o=\frac{E_i}{E_o}\,\frac{t}{n}\,\frac{x}{L}$$

The corresponding solutions for $E_o=E_i$ are shown in Fig. 4. For $t_i(L)>0$ and $C=0$, $t_o\to\infty$ as $x\to L$. This represents an impractical design and one for which one-dimensional membrane theory is not appropriate. However, for C as small as

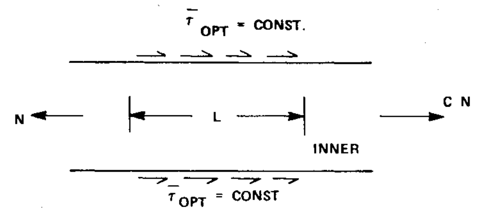

Fig. 2 Adhesive shear stresses for double lap optimum design.

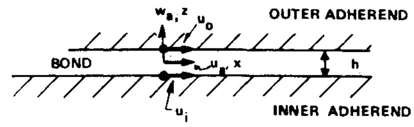

Fig. 3 Displacements and coordinate system used to describe motion of adhesive.

0.25, we obtain from Eq. (9)

$$t_o(L) = 3\frac{E_i t_i(L)}{nE_o}$$

which represents a design that is realistically achievable in many cases.

Nonuniform Material Case

When the inner material is not uniform (such as for composite materials), the method for determining $t_o(x)$ proceeds as follows: First, apply uniform shear stresses on the sides of the inner member where an adhesive will be used, and numerically compute the outer fiber strain distribution $u_i'(x)$. This may be easily done, for example, by using a standard finite element code. Since the adherend strains are related through Eq. (4), the outer member stress-strain law can be written as follows:

$$N_o = E_o t_o u_i' \tag{10}$$

Combining Eqs. (10) and (8) then yields the desired design relationship for $E_o t_o$, i.e.,

$$E_o t_o = \bar{\tau}_{\text{opt}} x / u_i' \tag{11}$$

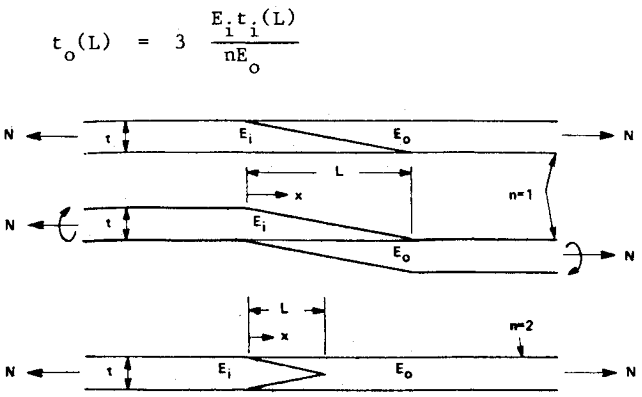

Fig. 4 Uniform material and uniformly tapered optimum bonded joints for single ($n = 1$) and double ($n = 2$) lap joints.

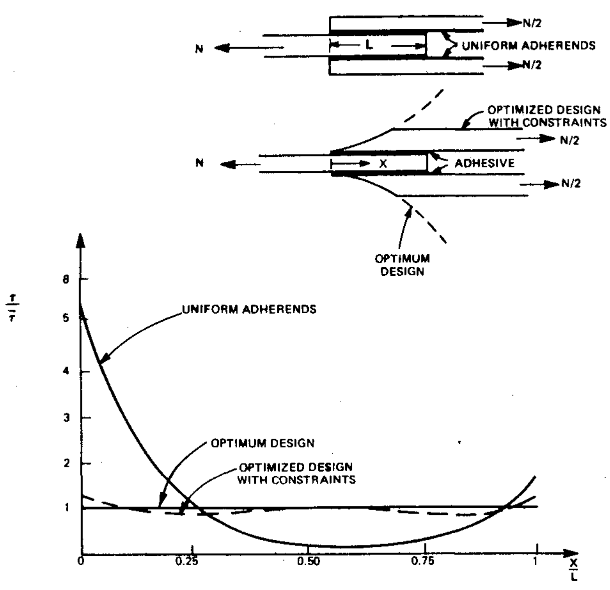

Fig. 5 Comparison of elastic adhesive shear stresses in double lap joints with uniform and optimized adherends.

Nonoptimum Considerations

Because of physical design constraints, it may be necessary to limit $t_o(x)$ to a maximum value smaller than is required to achieve a uniform adhesive shear stress. In addition, no consideration has been given to the adhesive peel stresses in the above design process. Such stress components, however, are required to achieve moment equilibrium in each outer adherend. Therefore, once a configuration has been developed by the methods suggested by Eqs. (9) and (11), it should be numerically analyzed to determine the actual shear and peel stress concentrations.

Another important design parameter that influences the actual stress concentration factor is the minimum value to which t_o may be reasonably machined. Equations (9) and (11) require that at $x = 0$, $t_o(0) = 0$. Although it is not possible to manufacture a perfectly sharp zero-thickness edge, we have found that an edge thickness of 0.001 or 0.002 in. will produce a modest shear stress concentration factor (of the order of 1.25) at $x = 0$. This is far lower than would occur with a typical nonoptimum design.

Table 1 Bond shear stress concentration factors for uniform and optimized double lap joints[a]

G_a/h, lb/in.^{3}, $\times 10^6$	λL, $\times 10^{-3}$	b/L	$\tau/\bar{\tau}$ $x=0$	$\tau/\bar{\tau}$ $x=L$
5	3.69	0[b]	2.77[b]	0.94
		0.500	1.19	—
		0.625	1.19	1.42
		0.750	1.17	1.14
10	5.22	0[b]	3.86[b]	1.37
		0.500	1.25	—
		0.625	1.28	1.46
		0.750	1.33	1.23
20	7.38	0[b]	5.44[b]	1.71
		0.500	1.49	—
		0.625	1.49	1.66
		0.750	1.54	1.30

[a]Joint parameters used: $t_o(0) = 0.015$ in., $(E_i t_i / 2E_o t_o) = 1/3$, $[L/t_o(b)] = 2.63$, $\lambda L = 0.165 \times 10^{-5}\sqrt{G_a/h}$.
[b]Nonoptimized uniform adherend cases.

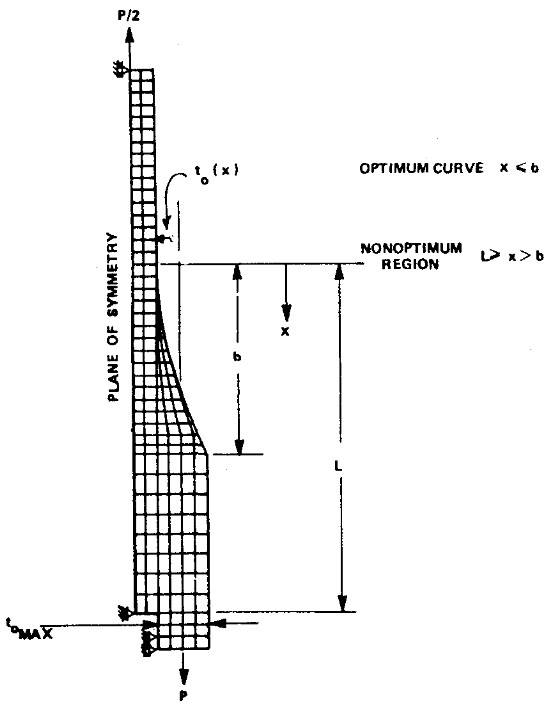

Fig. 6 Finite element model of half of symmetric double lap joint used to determine adhesive stresses.

An interesting and noteworthy feature of the derived solution is that the adhesive shear modulus G_a and bond thickness h do not enter into Eqs. (9) and (11). However, the ratio G_a/h does become important when nonoptimum conditions, such as those cited above, exist. In general, one can expect that stress concentration factors are reduced with decreasing values of this parameter.

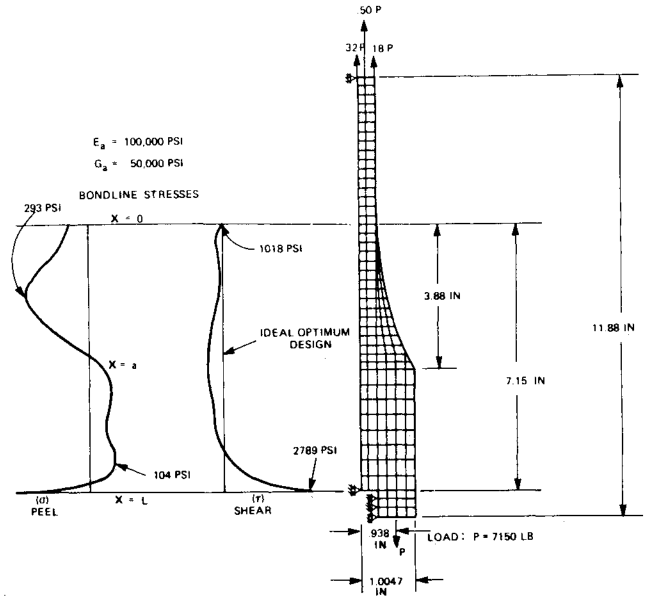

Fig. 7 Typical shear and peel stresses in partially optimized double lap joint.

Numerical Results—Uniform Inner Adherend

The typical elastic adhesive shear stress distribution for a double lap joint with uniform adherends is shown in Fig. 5. The stresses tend to peak at either edge, producing stress concentrations far in excess of those for the ideal optimum uniform stress state. These two extreme stress states are contrasted in Fig. 5, with the state shown by the present method, i.e., an optimized shape with the realistic constraints of a uniform inner adherend, a nonzero tip, and a maximum thickness that truncates the optimum curve.

The elastic shear stresses τ for the uniform adherends were obtained from the analytical solution derived by Hart-Smith[6]

$$\tau = A\sinh\lambda x + B\cosh\lambda x \tag{12}$$

where

$$\lambda^2 = \left(\frac{1}{E_o t_o} + \frac{1}{E_i t_i}\right)\frac{G_a}{h} \tag{13}$$

and the constants A and B are determined from the boundary conditions

$$\tau'(L) = \frac{N}{2}\,\frac{G_a/h}{E_o t_o} \tag{14}$$

and

$$\int_0^L \tau \mathrm{d}x = \frac{N}{2} \tag{15}$$

The elastic stresses for the optimized shape with thickness constraints were obtained from a finite element model of half of a symmetric double lap joint (see Fig. 6). Finite element models were also used to study the effect of varying parameters [such as optimum-shape truncation length b, tip thickness $t_o(0)$, and adhesive stiffness G_a/h] upon maximum shear stress.

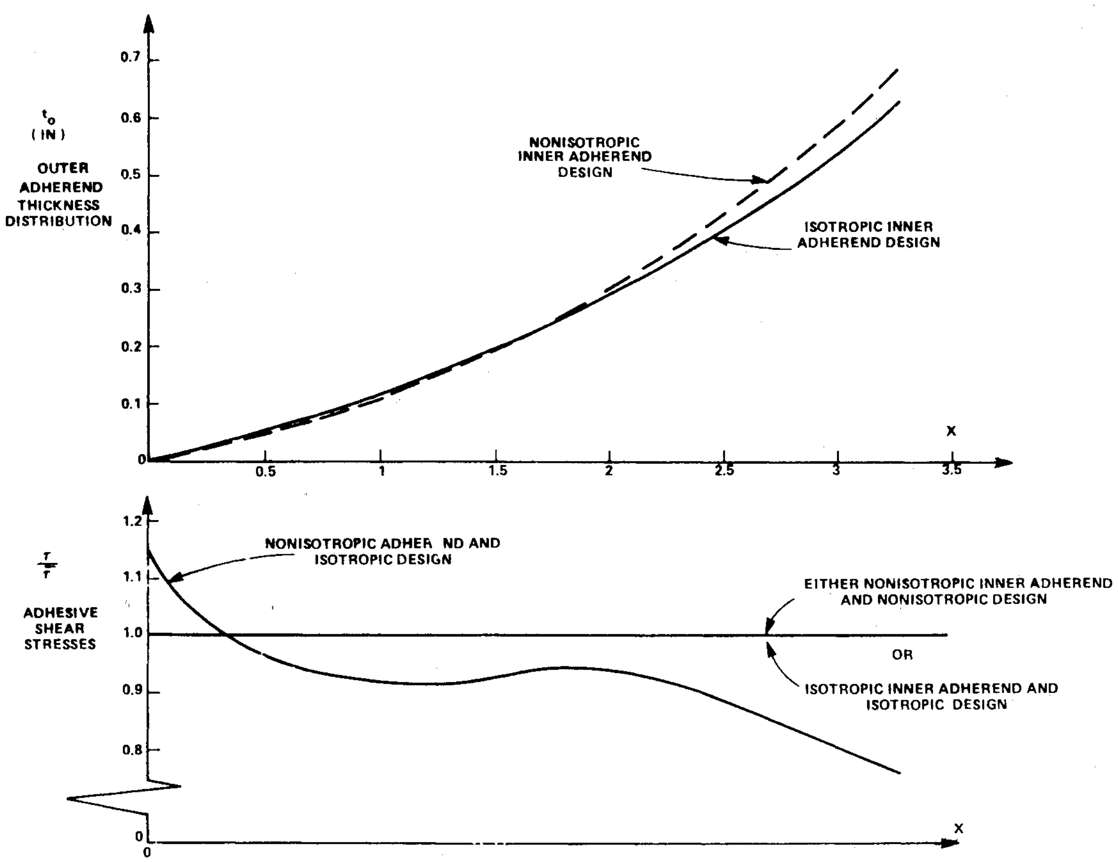

Fig. 8 Comparison of outer adherend thickness and stress distributions.

A typical peel stress distribution for the model depicted in Fig. 6 is shown in Fig. 7. As may be seen, the peel stresses for the optimized double lap joint are an order of magnitude lower than the shear stresses.

Furthermore, the shear stress at the nonoptimized end ($x=L$) peaks up and is much greater than the value at $x<b$, where the shape has been optimized.

The stress peaks at $x=0$ and $x=L$ for optimized joints, using various values of $t_{o\ \max}$, are shown in Table 1. The solution for a similar uniform section joint was obtained in closed form by the method described above, and numerical results are also presented in Table 1 (see $b/L=0$ cases).

We have also obtained numerical results for an inner material composed of varying layers of a graphite-epoxy composite inner web material using Eq. (11). The resulting optimum shape curve is compared in Fig. 8 with that for a uniform material with the same average stiffness properties. The results show how the use of a nonisotropic inner material can affect the optimum outer material shape and shear stress distribution if the shape is varied by as little as 5-10%. Additional design sensitivity runs have been made in which the thickness distribution was varied. These results indicate that a curve variation of over 10% begins to show a consequential increase in stress concentration at the joint edges.

It should be noted that the precise taper is critical and that the elastic stress results are sensitive to the specific taper design.

Discussion

The present design procedure works well for double lap joints when $b/L<0.625$, even though we have ignored bending in the design (but not the final numerical analysis) of the joint. This same approach was proposed by Cherry and Harrison,[4] who derived the single lap joint equivalent of Eqs. (9) and (11). They did not, however, explore the effects of joint eccentricity and optimum shape truncation upon shear and peel stress.

Our experience with the proposed optimum design procedure has shown that the shapes obtained produce peel stresses that are much lower than the shear stresses. Possible exceptions to this, however, can occur with single lap joints or if design constraints require a severe truncation ($b/L<0.5$) of the optimum curve. We have also observed that lower values of G_a/h tend to reduce stress concentrations in designs for which nonoptimum design constraints must be accommodated. This latter result is expected, since results for nontapered bonded joints also show a similar dependence upon adhesive stiffness G_a/h.

The main contribution of this paper is the presentation of a rational method for optimizing bonded joints. With regard to double bonded joints in which $C=0$ (where C is the load transfer ratio), $\tau/\bar{\tau}$ at $x=L$ (the nonoptimized edge) is of the same order as for the optimized edge $x=0$ (see Table 1). For $C>0$, Eq. (9) presents an infinity of optimized solutions.

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